Indian J. Phys. **75A** (6), 639-641 (2001)

IJP A
an international journal

On a problem of piezoelectric bar under electrical and thermal excitation

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Received 24 December 2000, accepted 10 August 2001

Abstract An attempt has been made to investigate analytically the mechanical behaviour of an open-circuited piezoelectric quartz bar, one end of which is subjected to some prescribed electrical and thermal excitations while the other end is kept fixed. The method of operational calculus has been used and the numerical results are illustrated graphically. For time-scale ranging from 0 to 1s variations of mechanical disturbances exhibit parabolic in nature and it is found to be of the order of 10^{-13} m.

Keywords Piezoelectric for electrical and thermal excitation, mechanical disturbances

PACS No. 77.65.-j

1. Introduction

The studies of piezoelectric transducers from the stand point of mechanics of continuous media have been initiated by a number of workers [1-5] *etc*. However, the problems in the thermo-piezoelectricity have been rarely considered in the literature. In view of various practical applications in different branches of physics and technology, the relevant problems are extremely important. Earlier workers like Sinha [6], Das [7], Gibbs [8], Sasaki and Takeuchi [9] *etc*. discussed the situations where two fields, *viz*. mechanical and electrical interact with each other. Certainly, the studies become more interesting if the above interaction is coupled with a thermal field. The present study is an attempt to this end and is a follow-up of the papers by Samoilov and Shchedrina [10], Paul and Raman [11] and Imano and Okuyama [12].

The present paper deals with a problem of mechanical behaviour of an open-circuited piezoelectric bar, one end of which is subjected to some prescribed electrical and thermal excitations while the other end is kept fixed. Since this problem involves the interaction of three fields, *viz*., electrical, mechanical and thermal, Maxwell's equation, equation of elasticity and the heat flow equation have been used and the solution is obtained with the aid of operational calculus. The variation of the mechanical disturbances with time is found to be parabolic in nature and is of the order of 10^{-13} m.

The present study is perhaps useful in the fields of acoustic signal processing, surface acoustic wave applications and acoustic delay lines.

2. The problem, fundamental equations and boundary conditions

We consider here an open-circuited piezo-quartz bar, one end of which is subjected to some prescribed electrical and thermal excitations while the other end is kept fixed. Our object is to obtain the mechanical response exhibited by the bar. We shall consider here transient input signal.

Since the problem involves the interaction of three fields, *viz*., mechanical, electrical and thermal, we must have an equation involving them. To derive such an equation, we take the relevant piezoelectric equations

$$T = c\,\partial\xi/\partial x - hD - \lambda\theta, \tag{1}$$

$$E = -h\,\partial\xi/\partial x + \beta D - \gamma\theta, \tag{2}$$

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where T, ξ and θ are stress, displacement and temperature in the x-direction respectively, c is the elastic stress co-efficient, h – the piezoelectric stress constant, β – the electromechanical coupling factor, λ – the thermoelastic compliance, γ – the thermo-piezoelectric modulus, D and E are the electric displacement and electric intensity respectively.

The equation of motion in the x-direction is given by

$$\rho \partial^2 \xi / \partial t^2 = \partial T / \partial x, \tag{3}$$

where ρ is the density of piezo-electric material.

The electric displacement D satisfies the equation

$$\operatorname{div} D = 0.$$

To obtain the equation for the displacement ξ, we must make some simplifying assumptions, *viz.*

(i) The dimension X of the bar is several times larger than Y or Z.

(ii) The X-Z faces of the bar are covered with conducting electrodes so that $\partial E / \partial x = 0$.

(iii) The X–Z faces of the bar are thermally insulated so that $\partial \theta / \partial x = 0$.

(iv) The end at $x = X$ is rigidly fixed.

Now from eq. (3) with the aid of eqs. (1) and (2) and the assumptions $\partial E / \partial x = 0$ and $\partial \theta / \partial x = 0$, we obtain the wave equation

$$\partial^2 \xi / \partial t^2 = \alpha^2 \partial^2 \xi / \partial x^2, \tag{5}$$

where $\alpha = \left(\beta C - h^2\right) / \rho \beta$.

Now, for this problem, the fundamental eqs. (1), (2) and (5) are to be solved subject to the following boundary conditions at $x = 0$ and $x = X$:

$$\text{(i)} \quad (\bar{\xi})_0 = \left(\bar{\xi}_1\right)_0,$$

$$\text{(ii)} \quad \left(\bar{F}\right)_0 = \left(\bar{F}_1\right)_0, \tag{6}$$

$$\text{(iii)} \quad \left(\bar{\xi}\right)_x = 0,$$

together with

$$\text{(a)} \quad V = V_0 \exp(-\omega t),$$

$$\text{(b)} \quad \phi = \phi_0 \exp(-\omega t), \tag{7}$$

$$(t > 0,\ \omega > 0)$$

where V and ϕ represent the electrical voltage and heat influx respectively and V_0 and ϕ_0 are constants.

3. Solution of the problem

Applying Laplace transform to eqs. (5) and (7), we obtain

$$d^2 \bar{\xi} / dx^2 = \left(p^2 / \alpha^2\right) \bar{\xi} \qquad (\operatorname{Re} p > 0) \tag{8}$$

and

$$\bar{V} = V_0 / p + \omega,$$

$$\bar{\phi} = \phi_0 / p + \omega. \tag{9}$$

The solution of eq. (8) is given by

$$\bar{\xi} = A \exp(-px/\alpha) + B \exp(px/\alpha), \tag{10}$$

where A and B are constants to be determined from the boundary conditions.

We may mention here that a wave equation of the form of eq (5) is satisfied even if the material be non-piezoelectric but with a different wave velocity. Therefore, we assume, after Redwood [4], that two mechanical systems labelled 1 and 2 are attached to the two extremities of the bar at $x = 0$ and $x = X$. The displacements in those materials will be similar to (10) with different values of A and B, say A_1, B_1 and A_2, B_2 in materials 1 and 2 respectively

To develop next the relation between electrical, mechanical and thermal quantities, we put, as in Sinha [6]

$$F = TYZ,\ V = EY,\ \phi = K\theta / Y$$

where F, V, ϕ represent the mechanical force, electrical voltage and heat influx respectively and K denotes the constant of diffusion.

With these substitutions, we obtained the required relation from eqs. (1) and (2) as

$$\bar{F} = p\alpha^2 YZ\rho\left[-A\exp(-px/\alpha) + B\exp(px/\alpha)\right]$$
$$-(hZ/\beta)\bar{V} - \{\lambda + (h\gamma)/\beta\}(Y^2 Z/K)\bar{\phi}. \tag{11}$$

From eqs. (6), (9), (10) and (11), we obtain the following relations

$$B_1 = A + B, \tag{12}$$

$$p\alpha^2 YZ\rho(-A + B) - (hz/\beta)\, V_0(p + \omega)$$
$$-\{\lambda + (h\gamma/\beta)\}(Y^2 Z/K)\,(\phi_0 / p + \omega) = p\alpha_1^2 Y_1 Z_1 \rho B_1, \tag{13}$$

$$A \exp(-pX/\alpha) + B \exp(pX/\alpha) = 0. \tag{14}$$

Solving eqs. (12) – (14), we obtain the values of A and B as

$$A = \exp\ (pX/\alpha)\ \mu\left[p(p+\omega)\{C_2 \exp(-pX/\alpha)\right.$$
$$\left.-C_1 \exp\ (pX/\alpha)\}\right]^{-1},$$

$$B = \exp\ (-pX/\alpha)\ \mu\left[p(p+\omega)\{C_1 \exp(pX/\alpha)\right.$$

$$\left. -C_2 \exp\ (-pX/\alpha)\}\right]^{-1},$$

where

$$C_2/C_1 = \left(\alpha_1^2 Y_1 Z_1 - \alpha^2 YZ\right)/\left(\alpha_1^2 Y_1 Z_1 + \alpha^2 YZ\right) = C_0 \text{ (say)} \quad (15)$$

and

$$\mu = (hZ/\beta)V_0 + \{\lambda + (h\gamma/\beta)\}\ (Y^2 Z/K)\phi_0 .$$

Substituting the values of A and B in eq. (10), we get,

$$\left(\bar{\xi}\right)_0 = (-\mu/C_1)\{1 - C_0 \exp(-2pX/\alpha)\}^{-1}$$

$$\times \{1 - \exp(-2pX/\alpha)\} 1/p(p+\omega), \quad (16)$$

where C_0 is given by eq. (15).

The inverse transform of eq. (16) is given by

$$(\xi)_0 \approx \left[\{1 - \exp(-\omega t)\} - (1 + C_0)\{H(t - 2X/\alpha)\right.$$

$$\left. -H(t - 4X/\alpha) + H(t - 6X/\alpha) - (1 - \exp(-\omega t)/w\}\right]$$

$$(-\mu/\omega C_1). \quad (17)$$

The eq. (17) gives out the mechanical disturbance of a piezo-quartz bar.

4. Discussion

For numerical calculation, the standard values of the numerical constants have been taken from [8-12] while values like X, Y, Z, ω, V_0, ϕ_0 have been chosen suitably to facilitate the numerical calculations as follows :

$X = 0.1$ m, $Y = Z = 0.01$ m, $V_0 = 300$ V, $\phi_0 = 4.18 \times 10^3$ J,

$\omega = 1.5$ rad/s.

$H(t) = 0$ for $t < 0$	$C_0 = 28.33$,	$h = 5.02 \times 10^4$,	$\beta = 0.03$,
$= 1$ for $t \geq 0$	$\alpha = 1$,	$\mu = 3800$,	$\rho = 2.56$.

The variation of mechanical disturbance with time is shown in the Table 1.

The response given out by piezo-quartz bar is illustrated in Figure 1. For time-scale ranging from 0 to 1 sec., variations of mechanical disturbances exhibit parabolic in nature and it is found to be of the order of 10^{-13} m. The response reduces to zero as $t \to 0$ and this treatment is valid only within the investigated range of time.

Table 1. Numerical values of the mechanical disturbances of piezo-quartz bar $(\xi)_0$ *vs* time (t)

t (sec)	$(\xi)_0 \times 10^{-13}$ (m)
0	0
0.1	- 1.25
0.2	- 2.40
0.3	- 3.45
0.4	4.40
0.5	- 5.25
0.6	- 6.05
0.7	- 6.75
0.8	7.25
0.9	7.85
1.0	- 8.25

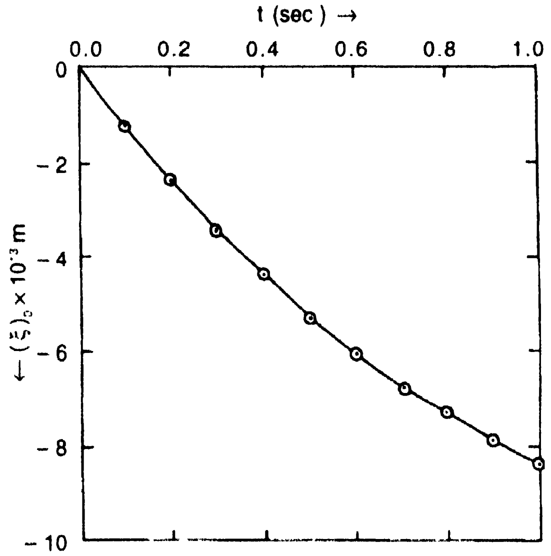

Figure 1. Variation of mechanical disturbance of a piezoelectric bar with time

References

[1] W P Mason *Piezoelectric Crystals and their Applications to Ultrasonics*, (New York : D Van Nostrand) p84 (1950)

[2] F J Haskins and L J Walsh *J Acoust Soc Am* **29** 729 (1951)

[3] W G Cady *Piezoelectricity* (New York : MC Graw Hill) (1959)

[4] M Redwood *J Acoust Soc Am* **33** 527 (1961)

[5] G Paria *J Sci Engg Res* **4** 381 (1960)

[6] D K Sinha *Indian J Mech. Math.* **2(1)** 10 (1964)

[7] N C Das *Indian J Theo Phys* **16(1)** 33 (1968)

[8] D F Gibbs *Ferroelectrics* (UK) **82** 133 (1988)

[9] Y Sasaki and Y Takeuchi *J Acoust Soc Jpn (E)* **10** 181 (1989)

[10] V B Samoilov and L V Shchedrina *Ferroelectrics* (UK) **118** 225 (1991)

[11] H S Paul and G V Raman *J Acoust Soc Am* **90** 1729 (1991)

[12] K Imano and D Okuyama *Trans. Inst Electron. Inf. Commun Engg* (Japan) **74A** 797 (1991)